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Integral Discrete-time Sliding Mode Control of Homogeneous Charge Compression Ignition (HCCI) Engine Load and Combustion Timing

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Abstract: Integral Discrete-time Sliding Mode Control (IDSMC) for combustion timing and load control of a single cylinder Homogeneous Charge Compression Ignition (HCCI) engine is designed and tested in simulation. First, a Nonlinear Control Oriented Model (NCOM) which is based on measurements is linearized around one operating point and the linearized model is then validated against the NCOM. The linearized model is then combined IDSMC with fueling rate and Exhaust Valve Closing (EVC) used as actuators. The controller is verified in simulation using the NCOM as a virtual engine. The simulation results indicate that IDSMC has acceptable performance for combustion timing and load control in HCCI engine and the performance is superior to classic PI control.

Keywords: HCCI Engine, IDSMC, VVT, Modeling, and Simulation

1. INTRODUCTION

HCCI is a promising Low Temperature Combustion (LTC) engine mode used to reduce particulate matter, NO_X and fuel consumption at low loads Zhao (2007). HCCI combustion is a hybrid of conventional Spark Ignition (SI) and diesel Compression Ignition (CI). In HCCI engines, lean premixed air-fuel mixture is compressed until combustion starts due to auto-ignition at many locations. It is difficult to control HCCI combustion timing, so the following ways have been proposed for HCCI combustion timing control: intake air heating Najt and Foster (1983); dual fuels Slepicka and Koch (2016); water injection Chistensen and Johansson (1999); Variable Valve Timing (VVT) Ravi et al. (2012) and Ebrahimi and Koch (2018b); Exhaust Gas Recirculation (EGR) Christensen and Johansson (2000); and variable compression ratio Christensen et al. (1999). HCCI combustion timing is mainly controlled by chemical kinetics of the trapped charge; therefore, the mixture composition, temperature and pressure at the Inlet Valve Closing (IVC) must be controlled for combustion timing control. VVT is a simple precise actuator used to set IVC conditions for HCCI combustion timing control as it reduces residual gas heat losses, and achieves fast cycle-by-cycle control response Ravi et al. (2012); Ebrahimi and Koch (2018b). HCCI Combustion timing control is necessary as it affects engine energy distribution, and emissions Ebrahimi and Koch (2018a). A brief summary of some HCCI control strategies developed in the literature is provided next.

In Bidarvatan et al. (2014), a coupled feed-forward Discrete Sliding Mode Controller (DSMC) is introduced for HCCI combustion timing and load control. Combustion timing is controlled by changing the auto-ignition properties of the fuel using two Primary Reference Fuels. Extremum seeking (ES) is used to tune the PI control gains for combustion timing control in Killingsworth et al. (2009). The proposed ES has the advantage of achieving both optimal set-point determination and controller gain scheduling. An H_2 optimal controller is developed based on a linear model in Shaver et al. (2009) for HCCI combustion timing and peak in-cylinder pressure control by modulating Exhaust Valve Closing (EVC) and Intake Valve Closing (IVC) timings. Model Predictive Control (MPC) is developed for HCCI combustion timing control in Bengtssonv et al. (2006) using two different actuators: dual fuel and VVT with variable IVC timing. MPC with VVT as the main actuator demonstrates more direct control of combustion timing compared to dual fuel approach. In Ebrahimi and Koch (2018b), MPC is designed based on a four-state physical model developed in Ebrahimi and Koch (2015) and the controller is implemented on a single cylinder HCCI engine for combustion timing and load control using symmetric Negative Valve Overlap (NVO) and fueling rate as actuators. The controller shows acceptable performance in tracking step changes in combustion timing and load while considering constraints on the actuators and outputs. An MPC is designed that is based on a five-state physical model Ravi et al. (2012) is implemented on a multi-cylinder HCCI engine in Erlien et al. (2012) for load and combustion timing control using split fuel injection and valve timing as main actuators. In Ritter et al. (2018), MPC is developed for combustion timing and load control of a Gasoline Compression Ignition (GCI) engine using NVO pilot injection, injection during compression and the NVO duration as main actuators. A linear parameter varying (LPV) black box model is developed by identifying the system over twenty one operating points. A look up table is used to first bring the system close to the desired values, then MPC is used to stabilize the system to the desired condition considering constraint on rate of pressure rise.

In this paper, Integral Discrete-time Sliding Mode Control (IDSMC) is developed and compared to a manually tuned PI controller for load and combustion timing control of a single cylinder HCCI engine in simulation. The controller robustness in the whole state space is achieved and the controller stability is shown based on Lyapunov stability analysis. The IDSMC is developed based on linearized version of the Nonlinear Control Oriented Model (NCOM) developed in Ebrahimi et al. (2016). Symmetric NVO with variable VVT is used for HCCI combustion timing control while the fueling rate is used for output work control. The crank angle of fifty percent fuel mass fraction burned, θ_{50}, is used as the cycle by cycle measurement of combustion timing. The NCOM simulation provides θ_{50} and load Ebrahimi et al. (2016) while in the experiment, measured cylinder pressure is needed. The controllers ability in rejecting engine fuel equivalence ratio and intake temperature disturbances with measurement noise is examined.

Table 1. Engine Operating Conditions

Parameter	Values	Parameter	Values
T_{IVC}	363.3 [K]	α	0.12 [-]
ϕ	0.29 [-]	θ_{50}	3.7 [CAD aTDC]
$m_f LHV_f$	0.36 [kJ]	n	825 [RPM]
θ_{EVC}	20 [CAD bTDC]	η_c	69.8 [%]

2. HCCI ENGINE MODELING

The NCOM model was paramerized using experimental data and is able to predict combustion timing, combustion efficiency, and load with acceptable accuracy Ebrahimi et al. (2016). The NCOM structure is shown in Fig. 1. The NCOM is linearized around the operating point listed in Table 1. The operating point is selected based on experimental data in Ebrahimi et al. (2016). The linear state space model is given by

$$
\begin{aligned}
x_{k+1} &= Ax_k + Bu_k + v_k \\
y_k &= Cx_k + Du_k + w_k
\end{aligned}
\tag{1}
$$

where x_k, y_k, and u_k are the model states, outputs,and inputs respectively. A, B, C and D are the state space matrices of system depend on the operating conditions where the model is linearized. The linear model states, inputs and outputs are

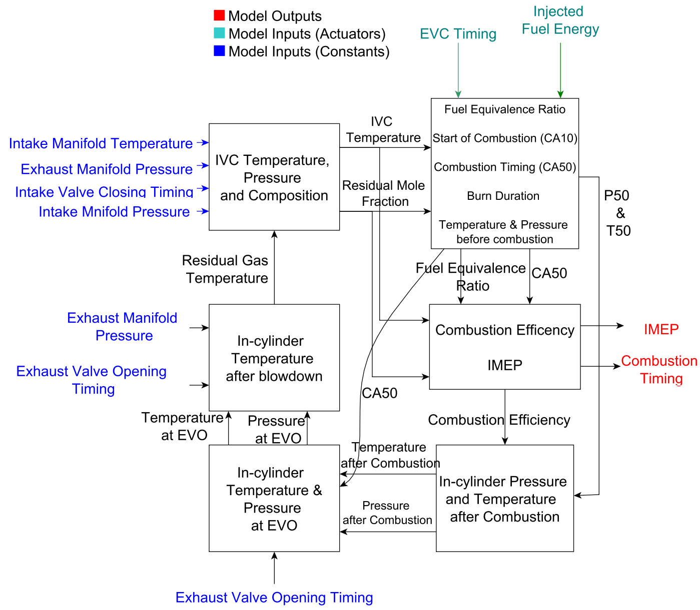

Fig. 1. Nonlinear Control Oriented Model Structure

$$
\begin{aligned}
x_k &= [\, T_{ivc} \quad \phi \quad \eta_c \quad \theta_{50} \,]^T \\
u_k &= [\, m_f LHV_f \quad \theta_{evc} \,]^T \\
y_k &= [\, \theta_{50} \quad IMEP \,]^T
\end{aligned}
\tag{2}
$$

where T_{ivc}, ϕ, η_c, θ_{50} are in-cylinder gas temperature at IVC, fuel equivalence ratio, combustion efficiency, and crank angle of fifty percent fuel mass fraction burned respectively. These states have important effects on HCCI engine combustion and performance Ebrahimi et al. (2016). The model inputs $m_f LHV_f$ and θ_{evc} are the injected fuel energy and EVC timing respectively. Finally, v_k and w_k are the state and output measurement disturbances defined as

$$|w_k| \leq \delta_y \tag{3}$$

$$|v_k| \leq \delta_x \tag{4}$$

To make matrix D in Eq.(1) zero, $m_f \times LHV$ and θ_{EVC} are added as new states to the linearized model. The resulting modified linear model is

$$
\begin{aligned}
x_{k+1} &= Ax_k + Bu_k + v_k \\
y_k &= Cx_k + w_k
\end{aligned}
\tag{5}
$$

where

$$
\begin{aligned}
x_k &= [\, T_{ivc} \quad \phi \quad \eta_c \quad m_f LHV_f \quad \theta_{EVC} \quad \theta_{50} \,]^T \\
u_k &= [\, m_f LHV_f \quad \theta_{evc} \,]^T \\
y_k &= [\, \theta_{50} \quad IMEP \,]^T
\end{aligned}
\tag{6}
$$

$$
A = \begin{bmatrix}
0.05950 & 26.41800 & 0.11080 & 0 & 0 & 1.29760 \\
-0.0000 & -0.0080 & -0.00003 & 0 & 0 & -0.0004 \\
0.01470 & 6.53700 & 0.02742 & 0 & 0 & 0.32110 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-0.0001 & -0.0391 & -0.0002 & 0 & 0 & -0.0019
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
-16.420 & -0.0180 & -10.320 & 0 & 1 & 0.06369 \\
0 & 0.67310 & 167.00 & 1 & 0 & -1.1490
\end{bmatrix}^T
$$

$$
C = \begin{bmatrix}
0.0006 & 0.2599 & 0.0011 & 16.79 & -0.4104 & 0.0128 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\tag{7}
$$

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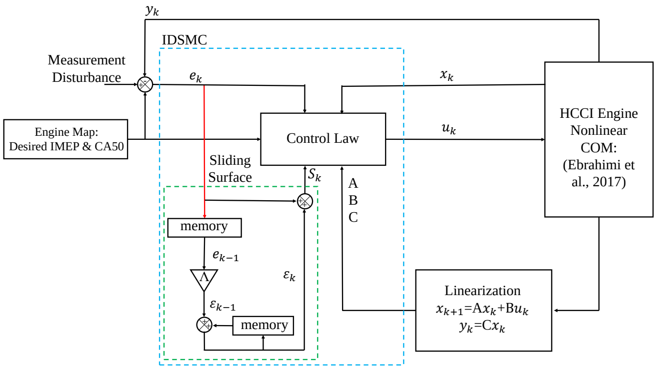

Fig. 2. The Block diagram of the proposed controller

3. CONTROL STRUCTURE

The controller is designed based on a discrete-time integral sliding surface. First, model uncertainty and measurement disturbances are considered in the integral discrete-time sliding mode controller. Then, the stability and switching gain margins are obtained based on the Lyapunov stability analysis.

3.1 Integral Discrete-time sliding Mode control (IDSMC)

Integral Discrete-time sliding surface is defined as

$$s_k = e_k - e_0 + \varepsilon_k \tag{8}$$

where s_k, e_k, e_0, and ε_k are the discrete time sliding surface function, output tracking error, initial tracking error, and the integral of output tracking error respectively. Defining

$$\varepsilon_k = \varepsilon_{k-1} + \Lambda e_{k-1} \tag{9}$$

$$e_k = y_{d,k} - y_k \tag{10}$$

where Λ is integral gain which is a strictly positive constant and $y_{d,k}$ is

$$y_{d,k} = [\,\theta_{50,d,k}\ \ IMEP_{d,k}\,]^T \tag{11}$$

where $\theta_{50,d,k}$ and $IMEP_{d,k}$ are the desired combustion timing and output work. The IDSMC control law is obtained based on the sliding surface (Eq. 8) as

$$u_k = (CB)^{-1}(\hat{u}_k - K sign(s_k)) \tag{12}$$

where K is the switching gain of IDSMC and $\hat{u}_k$ is

$$\hat{u}_k = (I - \Lambda)e_k + y_{d,k+a} - (CA)x_k - s_k \tag{13}$$

One of the challenges in the sliding mode control is chattering Slotine et al. (1991). Numerous methods have been proposed in literature to overcome chattering. In Slotine et al. (1991), both constant and variable boundary layers are introduced. Also, fuzzy logic can be used to modified constant boundary layer to deal with controller chattering Norouzi et al. (2018a). Additionally, some meta-heuristic optimization algorithms can be used for finding optimum boundary layer without system chattering Norouzi et al. (2018b). In this study a constant boundary layer based on a saturated function is used to eliminate controller chattering. Then, Eq. 12 is replaced by

$$u_k = (CB)^{-1}(\hat{u}_k - K sat(s_k/\mu)) \tag{14}$$

where μ is boundary layer thickness. The block diagram of the proposed controller is shown in Fig. 2. The controller is first designed based on the linear model then the controller is tested on simulation on the NCOM.

Proof. IDSMC control law (Eq. 14)

The sliding surface of integral sliding mode controller for a continuous system is define as Slotine et al. (1991)

$$s = (\frac{d}{dt} + \Lambda)^{n-1} \int edt \tag{15}$$

As the order of the engine model is 2 with respect to the integral of tracking error, $n = 2$ in Eq. 15, so Eq. 15 can be rewritten as

$$s = e + \Lambda \int edt \tag{16}$$

A forward difference method is used for discretization of the system. So, Eq. 16 in discrete-time is

$$s_k = e_k - e_0 + \varepsilon_k. \tag{17}$$

$$\varepsilon_k = \varepsilon_{k-1} + \Lambda e_{k-1} \tag{18}$$

The objective of IDSMC Xu and Abidi (2008) is to achieve $s_{k+1} = 0$. Now substituting k with $k+1$ into Eq. 17 and using $s_k + e_0 = e_k + \varepsilon_k$. an expression based on sliding surface is obtained. The state space linear model (Eq. 5) combined with the control Eq. 13 is

$$\begin{aligned} s_{k+1} &= e_{k+1} - e_0 + \varepsilon_{k+1} \\ &= Cx_{k+1} - y_{d,k+1} + \varepsilon_k - e_0 + \Lambda e_k \\ &= C(Ax_k + B\hat{u}_k) - y_{d,k+1} + S_k - (I - \Lambda)e_k \\ &= (CB)\hat{u}_k + (CA)x_k + S_k - (I - \Lambda)e_k \\ &\quad - y_{d,k+1} = 0 \end{aligned} \tag{19}$$

$$\hat{u}_k = (I - \Lambda)e_k + y_{d,k+a} - (CA)x_k - s_k \tag{20}$$

To satisfy the sliding condition sliding condition (Slotine et al. (1991)), a discontinuous term must be added to Eq. 20. Resulting in Eq. 12.

3.2 Stability analysis

A Lyapunov function is used to show the control stability. The derivative of Lyapunov function is negative, only if the switching gain of the proposed controller becomes bigger or equal to $\delta_y + |C|\delta_x + \eta$. Thus for stability

$$K \geq \delta_y + |C|\delta_x + \eta \tag{21}$$

where η is a strictly positive vector, δ_x and δ_y are upper bounds of disturbances value, and C is the linear state space model output matrix.

Proof. Stability Analysis and Gain margining

The candidate Lyapunov function is selected as

$$V = \frac{1}{2}s^2 \tag{22}$$

which is a Lyapunov function if and only if V is locally positive definite and its time derivative is locally negative semidefinite Khalil (1996). V is positive definite because s^2 has positive value. Substituting

$$\frac{d}{dt}V \leq -\eta|s| \tag{23}$$

where η is strictly positive vector. using Eq. 22 in Eq. 23 leads to

$$s\frac{d}{dt}s \leq -\eta|s| \tag{24}$$

which the discrete-time expression is

$$s_k(s_{k+1} - s_k) \leq -\eta|s_k| \tag{25}$$

The IDSMC controller is, based on the Lyapunov stability, stable if and only if Eq. 25 is fulfilled. Substituting Eq. 5 and Eq. 14 into Eq. 25 results in

$$\begin{aligned} s_{k+1} - s_k &= Cx_{k+1} + w_{k+1} - y_{d,k+1} \\ &\quad + s_k - (I - \Lambda)e_k - s_k \\ &= C(Ax_k + Bu_k + v_k) + w_{k+1} - y_{d,k+1} \\ &\quad + s_k - (I - \Lambda)e_k - s_k \end{aligned} \tag{26}$$

Thus, by applying the control law (Eq. 12) to this

$$\begin{aligned} s_{k+1} - s_k &= w_{k+1} + Cv_k + (CA)x_k \\ &\quad + (CB)((CB)^{-1}(\hat{u}_k - Ksign(s_k))) \\ &\quad - y_{d,k+1} - (I - \Lambda)e_k \\ &= w_{k+1} + Cv_k + (CA)x_k + (I - \Lambda)e_k \\ &\quad + y_{d,k+1} - (CA)x_k - s_k - Ksign(s_k) \\ &\quad - y_{d,k+1} - (I - \Lambda)e_k \end{aligned} \tag{27}$$

and simplifying using the triangle inequality, results in

$$\begin{aligned} & s_k(w_k + Cv_k) - s_k^2 - K|s_k| \\ & \leq s_k(w_{k+1} + Cv_k) - K|s_k| \\ & \leq |s_k||(w_{k+1} + Cv_k)| - K|s_k| \\ & \leq |s_k||w_{k+1}| + |s_k||C||v_k| - K|s_k| \\ & \leq -\eta|s_k| \end{aligned} \tag{28}$$

As $|s_k|$ is positive, it can be removed from both sides resulting in

$$\delta_y + |C|\delta_x - K \leq -\eta \tag{29}$$

Therefore, by letting $K \geq \delta_y + |C|\delta_x + \eta$, the stability of IDSMC controller has been satisfied.

4. RESULTS AND DISCUSSION

The linear model is validated against the NCOM and the results are shown in Fig. 3. The linear model captures the NCOM dynamics with acceptable accuracy. The IDSMC performance is compared to the manually tuned PI controllers and the results are shown for combustion timing and load trajectory tracking in Fig. 4. Both controllers are able to track the desired load and combustion timing. However, the PI controller cannot track desired combustion timing between cycles 180 and 275. The PI controller gains could be tuned to zero the steady state error for that range, however, gain scheduling is not performed to keep the PI controller structure simple. Gaussian distributed noise with standard deviations of 0.49 CAD and 0.057 bar are applied to the combustion timing and load. These values are chosen based on experimental measurement. The controller performance is checked for the desired combustion timing and load tracking in Fig. 5. Also, disturbances are applied to the ϕ, and T_{int} and the results are depicted in Figs. 6-7. These results indicate that IDSMC has excellent performance in tracking of the desired outputs, and the controller has better performance compared to the PI specifically when state disturbances and the measurement noise are considered. As shown in Figs. 6 and 7 the PI controller of θ_{50} is slow and is unusable respond to disturbance. To compare PI controller to IDSMC, Root Mean Square (RMS) of the errors are calculated based on. The RMS of $IMEP$ and θ_{50} errors for both IDSMC and PI controller are listed in Table 2. The minimum (Relative Percent Difference) RPD RMS of error is 35.4 percent; quantifying better the IDSMC performance compared to the PI. The IDSMC also has these advantages: the system

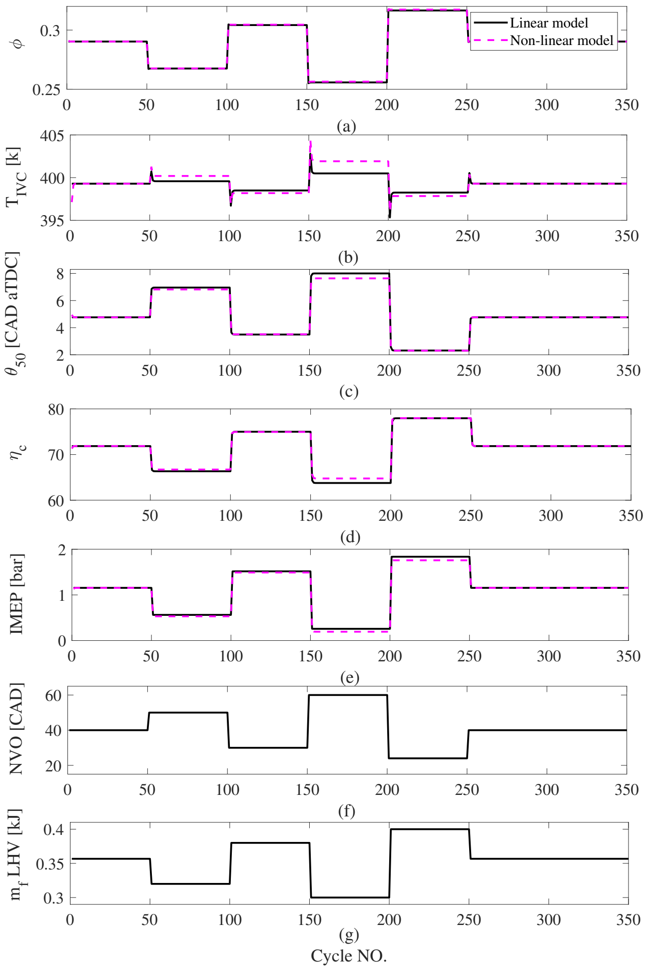

Fig. 3. Linear COM versus non-linear COM: (a) fuel equivalence ratio (b) temperature at intake Valve Closing (c) combustion timing (d) Combustion efficiency (e) load (f)negative valve overlap duration (g) injected fuel energy

trajectory always starts from the sliding surface; reaching mode during which the system is sensitive to noise in normal sliding mode that is eliminated; robustness in the whole state space is satisfied Utkin (1992); uncertainty and disturbance could be considered directly in controller design and stability analysis; and the stability of controller is shown based on Lyapunov stability analysis.

Table 2. RMS of errors comparison between PI and IDSMC

mode	Error type	PI	IDSMC	RPD
Without	RMS(E_{CA50})	0.765	0.260	65.9%
Disturbance	RMS(E_{IMEP})	0.106	0.059	44.7%
Disturbance	RMS(E_{CA50})	0.927	0.565	39.0%
on output	RMS(E_{IMEP})	0.116	0.079	35.4%
Disturbance	RMS(E_{CA50})	0.928	0.264	71.5%
on state	RMS(E_{IMEP})	0.116	0.059	49.4%
Disturbance	RMS(E_{CA50})	0.781	0.287	63.3%
on T_{int}	RMS(E_{IMEP})	0.107	0.060	43.5%

5. CONCLUSIONS

An Integral Discrete-time, Sliding Mode Control (IDSMC) strategy is developed based on a linear model for combus-

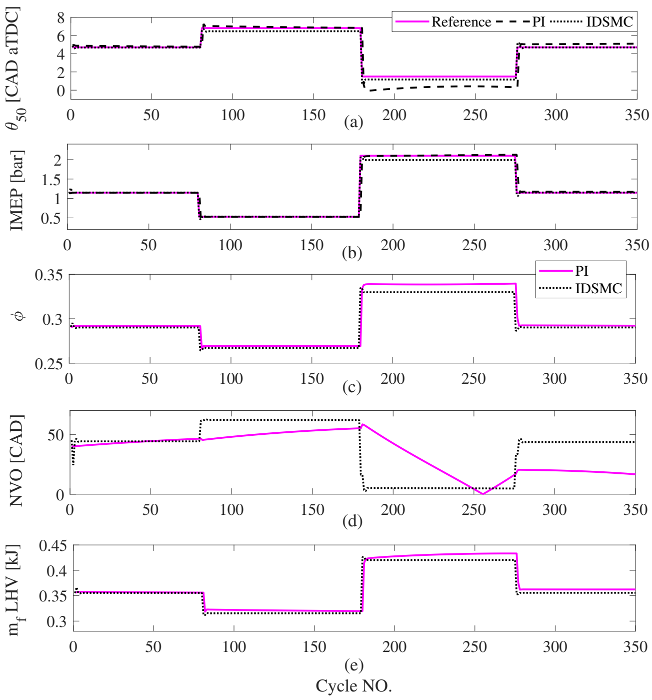

Fig. 4. Reference Input Tracking IDSMC versus PI (Simulation): Controller performance (a) combustion timing (b) load (c) fuel equivalence ratio (d) negative valve overlap duration (e) injected fuel energy

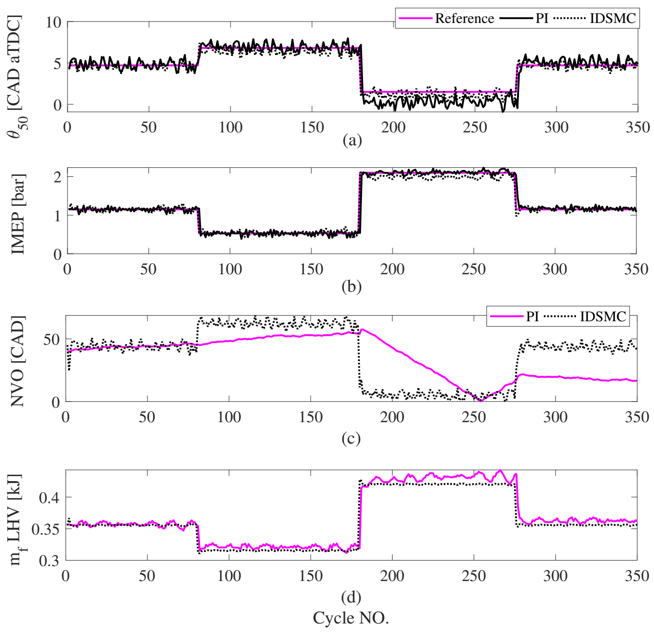

Fig. 5. Reference Input Tracking with sensor noise IDSMC versus PI (Simulation): Controller performance with output measured disturbance (a) combustion timing (b) load (c) negative valve overlap duration (d) injected fuel energy

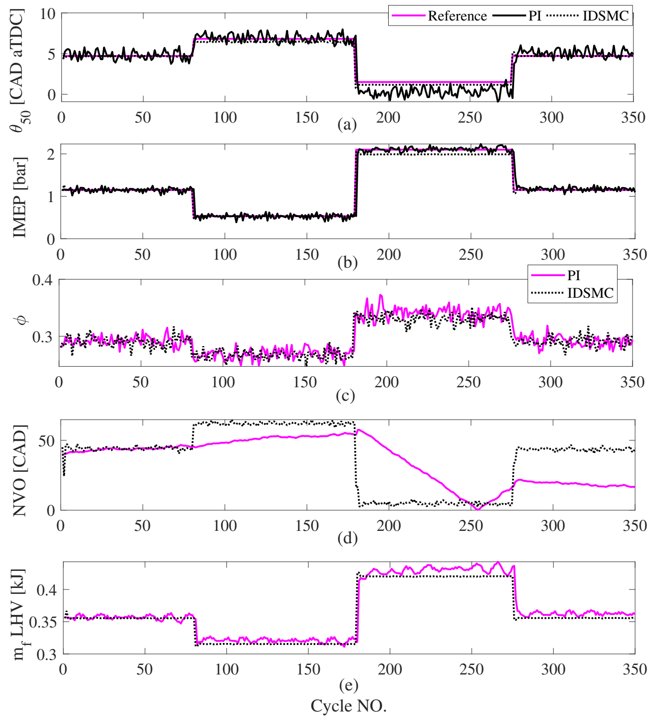

Fig. 6. Disturbance rejection IDSMC versus PI (Simulation): Controller performance with ϕ disturbance (a) combustion timing (b) load (c) fuel equivalence ratio (d) negative valve overlap duration (e) injected fuel energy

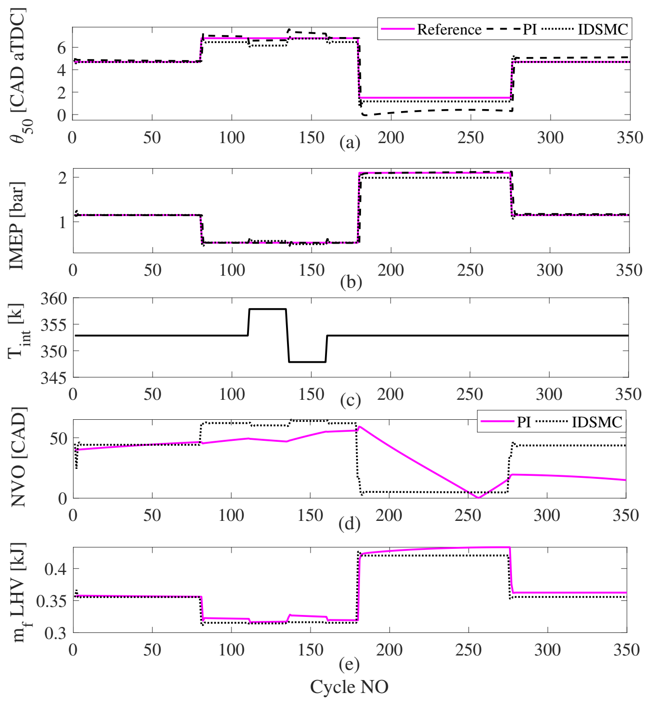

Fig. 7. Disturbance rejection IDSMC versus PI (Simulation): Controller performance with T_{int} disturbance (a) combustion timing (b) load (c) Intake Temperature (d) negative valve overlap duration (e) injected fuel energy

tion timing and load control of a single cylinder Homogeneous Charge Compression Ignition (HCCI). An existing validated Nonlinear Control Oriented Model (NCOM) is linearized around one operating point. The linear model is then validated against the NCOM for step changes in valve timing and fueling rate. The proposed controller is tested in simulation on the NCOM and the controller performance is compared to a classic PI controller. The IDSMC performance in tracking desired combustion timing and load is detailed. The IDSMC performs well in rejecting fuel equivalence ratio and intake temperature disturbances compared to PI controllers. The proposed IDSMC also shows better tracking accuracy compared to the PI controller in the presence of state disturbance and input uncertainties. To quantify the difference of the IDSMC compared to PI control, RMS error is calculated. The minimum difference of relative percent difference between the PI and the IDSMC controller is 44.7 percent for the conditions examined. The tracking control accuracy of IDSMC is 44.7 percent higher compared the PI controller. Both the nonlinear control oriented model and IDSMC have a simple structure and can be easily parametrized for other fuels such as bio-fuels.

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